Optimization of Rotor Performance in Hover Using a Free Wake Analysis

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Performance optimization for rotors in hover is a topic of continuing importance to rotorcraft designers. The aim of this research was to demonstrate that a free wake hover performance prediction code could be coupled to a numerical optimization algorithm. The hover code, dubbed EHPIC (evaluation of hover performance using influence coefficients), uses a quasilinear wake relaxation to solve for the rotor performance and is well suited to optimization applications. The coupling was accomplished by expanding the matrix of influence coefficients in EHPIC to accommodate design variables and by deriving new coefficients for linearized equations governing perturbations in power and thrust. These coefficients formed the input to a numerical optimization analysis, which used the flow tangency conditions on the blade and in the wake along with specified inequality constraints to constrain the design. It was found that this linearized analysis could be invoked to predict a design change that would produce significant reductions in power required at constant thrust. Thus, improved versions of the baseline design can be found efficiently while retaining the accuracy inherent in a free wake performance analysis. Sample problems were undertaken to demonstrate the success of this approach in reducing the power required at a specified thrust for several representative rotor configurations.

Nomenclature

$P_{()}$ = member of coefficient vector in the linearized power equation
$Q_{()}$ = influence coefficient submatrices
$T_{()}$ = member of coefficient vector in the linearized thrust equation
ΔP = perturbation in rotor power (objective function), N-m/s
$\Delta \boldsymbol{q}$ = vector of perturbations in crossflow velocities at wake collocation points, m/s
ΔT = perturbation in rotor thrust, N
$\Delta \boldsymbol{w}$ = vector of perturbations in downwash velocities at blade control points, m/s
$\Delta \boldsymbol{x}$ = vector of perturbations in wake collocation point positions, m
$\Delta \boldsymbol{\gamma}$ = vector of perturbations in bound circulation, m^2/s
$\Delta \boldsymbol{\theta}$ = vector of perturbations in blade design (typically, blade twist in radians)

Introduction

THE principal objective of this effort was to demonstrate the feasibility of developing a flexible and efficient performance optimization analysis for rotorcraft incorporating the influence coefficient/free wake approach. As will be described later, the results obtained largely fulfilled this objective and simultaneously provided insight into several design strategies for improving hover performance. The effort described here is best characterized as a technology demonstration because it was undertaken as the first phase in the development of a comprehensive optimization analysis. Although the results of the sample problems addressed here have been promising, the current capabilities of the analysis must be extended before it can become a generally applicable tool for rotorcraft performance optimization.

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In particular, it should be noted at the outset that the focus of this initial effort was on demonstrating that significant improvements in rotor performance could be obtained using this new approach. Current results suggest that the capability developed here represents a significant step on the road toward identifying truly optimized designs. The next two sections will discuss the background information on the problems of interest here as well as on the methods used to attack them in this effort. This will set the stage for the description of the sample problems addressed here and for recommendations for future work on this topic.

Background Information on Hover Performance Optimization

A prerequisite for addressing the requirements of rotorcraft designers for a design optimization capability is a reliable, generally applicable performance prediction program for hovering rotors. Without an accurate open-loop analysis, any attempt to optimize the rotor performance through design changes is of limited usefulness. Even with such a capability in hand, successive trial and error efforts to find a detailed design (by varying chord, twist, taper, and other layout parameters) to meet performance specifications is inevitably a time-consuming undertaking. If a purely heuristic process is used, promising designs may be missed, even with systematic parametric studies. Therefore, although accurate computational tools for predicting hover performance are essential for guiding efforts and abbreviating cut-and-try work with expensive subscale hardware, they themselves are used to best advantage when they are guided by formal optimization procedures that take advantage of numerical optimization routines.

The first priority, that of developing computational tools to analyze reliably hover performance for specified configurations, has been under study for well over 20 years (e.g., see Refs. 1–6). It has long been clear that the preferred approach in this area is a method involving the explicit computation of the free vortex wake of the rotor; this approach avoids reliance on empirical wake trajectories and yields force-free wakes, which in turn enhances confidence in performance predictions. The discussion that follows will summarize previous work in the development of free wake analyses as well the recent development of a new approach to the problem based on an influence coefficient formulation. As is implied earlier, though, the need for accuracy must be balanced against the requirement for a numerical optimization analysis that can efficiently evaluate promising changes in the baseline design with minimial use of the computationally expensive open-loop performance analysis.

The discussion below will outline the advantages of applying a free wake/influence coefficient analysis to the optimization problem. This analysis, dubbed EHPIC (evaluation of hover performance using influence coefficients), has proven to be a promising foundation for a free wake optimization routine, because it has produced good performance correlation for a range of rotor configurations and because its formulation permits straightforward evaluation and exploitation of information on the gradients in performance due to design changes. Although free wake analyses inevitably carry a computational penalty relative to simplified wake models, the results obtained here indicate that an optimization analysis based on the influence coefficient approach can perform efficient exploration of alternative designs without sacrificing the refined physical model associated with free wake treatments.

Early efforts to develop free wake hover models[5,6] were hampered by long computation time and poor convergence. More recently, a new approach to the free wake problem was devised[7,8] that incorporates curved vortex elements[9,10] as an aid to computational efficiency, in addition to a new wake relaxation scheme. This new approach solves for the free vortex geometry while circumventing the well-documented convergence problems associated with time-marching simulations through the use of an influence coefficient approach. As is described in detail in Refs. 8 and 11, the EHPIC code has produced very satisfactory performance predictions for a variety of rotor designs.

The general objective of a free wake hover analysis is to find the wake geometry that, for a specified rotor design, satisfies two conditions: first, that the wake filaments are in free motion; and second, that the flow tangency condition is satisfied on the blade. To achieve the free motion condition, the wake filament trajectories must be tangent to the local velocity vector evaluated on the filament when viewed in a rotating reference frame, i.e., there must be no crossflow velocity components at any point on the filaments under force free conditions. Initial estimates of blade loads and wake geometry will not, in general, satisfy the required conditions, and so must be adjusted in a succession of solution steps. To accomplish this, the bound circulation at stations along the blade and the vortex wake position coordinates are systematically perturbed, and the effect of these perturbation on the downwash on the blade and crossflow velocities in the wake are summed and formed into influence coefficients. These coefficients allow the construction of a set of linear equations in matrix form that predict the change in dependent variables due to the perturbations in independent variables. The influence coefficient array appears in a linear system of equations in the following form:

$$\begin{bmatrix} \Delta \boldsymbol{q} \\ \Delta \boldsymbol{w} \end{bmatrix} = \begin{bmatrix} Q_{qx} Q_{q\gamma} \\ Q_{wx} Q_{w\gamma} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{x} \\ \Delta \boldsymbol{\gamma} \end{bmatrix} \tag{1}$$

These variables are perturbations about an initial guess, which, in general, features some nonzero residual velocities on the left side. Were the problem purely linear, inverting this matrix and multiplying it by the residual velocity vector would yield a vector of wake position and circulation perturbations that would exactly null the residual velocities:

$$\begin{bmatrix} \Delta \boldsymbol{x} \\ \Delta \boldsymbol{\gamma} \end{bmatrix} = -\begin{bmatrix} Q_{qx} Q_{q\gamma} \\ Q_{wx} Q_{w\gamma} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{q}_0 \\ \boldsymbol{w}_0 \end{bmatrix} \tag{2}$$

In general, the process must be repeated due to the inherent nonlinearity of the problem, and only fractions of the residual velocities are nulled in each iteration. In practice, this approach has been found to have satisfactory convergence properties, despite the complexity of the relaxation procedure.

In the EHPIC code, the wake vortex filaments are represented by curved vortex elements, which provide an efficient means to perform the Biot-Savart integrations necessary for the evaluation of wake-induced velocities. A vortex lattice/lifting surface analysis is used to evaluate the thrust and induced torque on the rotor. Each blade is modeled by such a lattice, and the solution method produces an array of bound circulation values that null the downwash at each control point. Using the local values of freestream and induced velocity, the Joukowski law is used to find the force and moment on each lattice element. This procedure also produces a spanwise lift coefficient distribution that is used, along with the local Mach number, in a lookup scheme that computes the profile drag coefficient of that section. Reference 8 and the references therein contain additional discussions of the structure of the EHPIC analysis and its sensitivity to lattice and wake layout.

One advantage of the influence coefficient approach is that it finds the physically correct, self-preserving wake geometry without the instabilities and consequent lack of convergence of earlier methods that convected the wake in a time-marching manner (e.g., see Refs. 5 and 6). Furthermore, once a converged solution has been obtained, adjacent solutions along a performance curve are obtained readily by altering collective and reattaining equilibrium with a few (three or four) relaxation steps, thus eliminating the need to wash out the transients occurring in time-marching schemes, a process that is generally computationally much more expensive. This feature of the analysis is particularly useful from the point of view of optimization applications.

In the EHPIC code, the bound circulations and downwash at the converged state are used to evaluate the power required for a given thrust. The influence coefficient array that exists for the converged solution constitutes a linearized model of the downwash and the crossflow at that point. At the outset of the current effort, it was postulated that a suitably expanded version of this array could be used to explore the effects of changes in the rotor configuration on the performance. The first step in carrying this out was to calculate the influence coefficients associated with the introduction of design variables as new degrees of freedom. The expectation was that the linearized treatment of coupled blade/wake model could serve as the foundation of a design optimization analysis and would retain a refined aerodynamic model of the wake that was absent from previous work on this topic. Before proceeding further with the description of the development of this analysis, it is appropriate to briefly review previous efforts on rotor performance optimization.

There exists a substantial body of literature on numerical optimization procedures that are suitable for guiding design changes aimed at achieving specific performance goals. Recent reviews of aerospace- and rotorcraft-oriented applications of numerical optimization[12-18] give evidence of the maturity of these techniques. Following the early work of Stepniewski and Kalmbach[15] in the application of optimization techniques to rotorcraft, a variety of efforts were undertaken to attack the problem of rotor performance optimization using numerical techniques. Moffitt and Bissell[18] undertook a general examination of rotor airloads in both hover and forward flight, using a prescribed wake aerodynamic model with circulation coupling, with the aim of finding airload distributions that led to a minimum power required for a specified thrust. Nagashima and Nakanishi[19] studied hover performance optimization for coaxial rotors, employing both a closed-form "generalized momentum" model of the wake as well as a simplified free wake model based on vortex rings. Chung[20] also used a simplified free wake model to attack the formal optimization problem for hover performance. Finally, Walsh et al.[21] describe the assembly of several existing aerodynamic and vehicle trim models and a commonly available numerical optimization routine into a general blade design optimizer for hover and forward flight.

This previous work represents a broad range of accomplishment in the area of performance optimization, although these efforts have generally used simplified aerodynamic models such as strip theory or wake prescription. Such models contribute to the development of computationally efficient tools, a pressing concern in rotary wing aerodynamics. However, now that more computational power is becoming routinely available to analysts at all levels in the rotorcraft field, more advanced methods should be considered. Simplified models lack the generality and accuracy of more refined treatments featuring free wake models, and their use in an optimization analysis tends to diminish confidence in the true optimality of the final solution. A fundamental assumption of the current effort is that a free wake analysis like the one embodied in the EHPIC code is required to provide a suitable foundation for an advanced optimization treatment. Further developments in the solution of the Euler or Navier-Stokes equations for rotorcraft applications may ultimately provide a more general basis for design optimization, but the analysis developed here was judged to represent a reasonable balance of sophistication and efficiency for near-term application.

Outline of the Optimization Solution Procedure

To modify influence coefficient approach to hover performance in order to address the optimization problem it is necessary not only to introduce design variables into the solution method but also to select an objective function and appropriate constraint equations to ensure that the design problem is well posed. Both linear and nonlinear algorithms are available for application to the problem. Since the governing equations for the hover performance problem are in fact highly nonlinear, the appeal of applying one of several existing nonlinear programming algorithms to this problem was quite strong. However, linear optimization routines have an advantage in that they are, in general, more robust than their nonlinear counterparts; if a fundamentally nonlinear problem can be treated adequately by a sequential linear optimization, then the application of a linear optimization analysis becomes quite attractive. Though not an optimization calculation, the success of the linearized influence coefficient approach to the nonlinear free wake problem in hover was an example of the potential of just such an approach. It was thus judged appropriate to rely on a linear optimization routine for this effort, with successive local linear models of the coupled blade/wake system obtained using methods similar to the influence coefficient evaluation techniques embodied in the EHPIC code.

The details of the particular linear optimization routine selected for this effort are summarized briefly later and are discussed in more detail in Ref. 22. The objective function selected for use here was the total power consumed by the rotor in hover, and the design problem addressed was the minimization of power required at a specified thrust. Although, in general, any design variable could have been introduced (e.g. chord, sweep, etc.), for this effort the twist distribution on the rotor blade was selected. The equations governing the objective function and the thrust constraint then are, respectively,

$$\Delta P = [P_x]\{\Delta \boldsymbol{x}\} + [P_\gamma]\{\Delta\gamma\} + [P_\theta]\{\Delta\boldsymbol{\theta}\} \qquad (3)$$

$$\Delta T = [T_x]\{\Delta \boldsymbol{x}\} + [T_\gamma]\{\Delta\gamma\} + [T_\theta]\{\Delta\boldsymbol{\theta}\} \qquad (4)$$

where θ represents the blade twist distribution. In order to ensure that changes in blade design still lead to a coupled blade/wake system that satisfies the required flow tangency conditions, additional constraints are applied in the form of an expanded version of Eq. 1, i.e.,

$$\begin{bmatrix} \Delta \boldsymbol{q} \\ \Delta \boldsymbol{w} \end{bmatrix} = \begin{bmatrix} Q_{qx} Q_{q\gamma} Q_{q\theta} \\ Q_{wx} Q_{w\gamma} Q_{w\theta} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{x} \\ \Delta\gamma \\ \Delta\boldsymbol{\theta} \end{bmatrix} \qquad (5)$$

If the perturbations on the left side are set to zero, these equations can be treated as constraints on the changes in the wake position, the bound circulation, and the design variables.

As may be inferred from the form of problem posed here, the solution method developed for the EHPIC code provided a natural foundation for the formulation of the optimization problem. Most of the coefficients in the constraint relationships summarized by Eq. (5) are computed by the open-loop EHPIC code, as indicated by Eq. (1), and the calculation of additional coefficients was relatively straightforward. The calculation of the coefficients in Eqs. (3) and (4) was considerably more involved but was greatly simplified by the existence of the linearized velocity derivatives in Eq. (1).

The primary criteria in selecting the optimization algorithm itself were robustness and efficiency. The simplex linear optimization scheme fits both of these criteria. The particular algorithm used here was essentially identical to a version available in the open literature and proceeds to maximize the objective function $-\Delta P$, where ΔP is defined by the linearized function in Eq. 3. (This corresponds to finding the maximum decrease in power.) This algorithm also permits both equality and inequality constraints to be imposed on the perturbation variables in the problem. The inequality constraints can place user-defined limits on the total magnitude of the design variables. The equality constraints implicit in the form of Eq. 5 are augmented by the thrust constraint and inequality constraints on the changes in the wake position, the bound circulation, and the design variables. The details of the operations of the optimization algorithm are described more fully in Ref. 22.

Implementation of the Performance Optimizer

The coupling of the optimization algorithm to the EHPIC code was simplified by some of the inherent characteristics of the influence coefficient approach to the free wake problem in hover. As was described in Ref. 8, the EHPIC code has the ability to restart its solution process from a converged result with a small change in the collective pitch. A perturbation in pitch forces the analysis to reinvoke the quasilinear EHPIC relaxation several times to reattain equilibrium. This capability has been used extensively in early applications of EHPIC to generate new solutions for several adjacent collective settings without a complete reinitialization of the solution. This process can be streamlined considerably by taking advantage of the capabilities of the linear optimization algorithm. For example, the optimization routine just described produces predictions not only of the design changes needed to reduce power but also of the wake geometry and bound circulation perturbations that are compatible with these changes, given the constraints of constant thrust and the free wake boundary conditions. The initial computation of all of these perturbations is carried out immediately following the completion of the initial convergence of the baseline solution. The design perturbations must, of course, be added to the baseline design, but to accelerate the reconvergence of the solution, the perturbations in wake position and bound circulation are added on as well. This has the effect of moving the complete coupled system very close to its proper free wake equilibrium so the elimination of any residual crossflow or downwash velocities can be achieved with only one or two further calls of the EHPIC relaxation procedure.

The sequence of events in the current version of the optimization analysis is depicted in the flowchart in Fig. 1. For the purposes of most of the demonstration calculations dis-

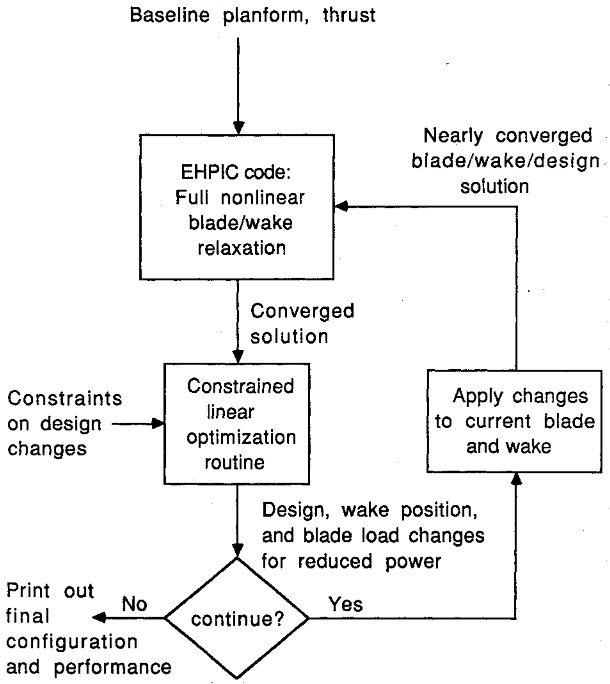

Fig. 1 Flowchart for the optimization analysis.

cussed, the analysis was permitted to run through as many loops as required to produce significant performance improvements; increases of several points in figure of merit at constant thrust were routinely observed. Sample calculations showing the effect of constraints on the total magnitude of the design variables are also described later. The flowchart also shows the necessity of specifying the constraints on the perturbations in independent variables that are to be permitted each time the optimizer is invoked. No hard-and-fast rules were developed here for setting the magnitude of allowable perturbations, although previous experience with the EHPIC code was helpful for guiding the choices for this effort. The maximum twist change considered permissible for any single segment in a given optimization step was typically 0.1 deg. Numerical experimentation further indicated that simultaneous constraints of roughly 0.01R on Δx and 2–3% of the mean bound circulation on $\Delta \gamma$ were appropriate choices.

Performance Improvements in Sample Problems

As noted earlier, for the purposes of this technology demonstration, the design variable chosen for the sample problems was the built-in twist of the rotor blade. The formulation of the problem to this point has been sufficiently general so that other variables (e.g., chord, sweep, etc.) could have been chosen. One of the requirements for a more complete follow-on analysis would be to explicitly accommodate any such variable. The selection of the twist allowed the best advantage to be taken of the previous work while still providing a valid demonstration of the capability developed here.

In this analysis, the blade layout is specified by selecting the twist, chord, sweep, and anhedral values at the edge of up to 10 separate segments on the span of the blade. Each quantity is assumed to vary linearly between the boundary values specified at the segment edges, so the analysis can generate a 10-segment piecewise-linear fit to any nonlinear design distribution. In the examples that follow, the collective pitch (i.e., the geometric pitch at the blade root) was held fixed while the segments outboard were allowed to distort. For most of the sample calculations discussed later, a representative "generic" rotor blade layout was used. This design featured untapered blades of radius 15 ft (4.6 m), chord 1.0 ft (0.30 m), 20% cutout, and a tip speed of 700 fps (213 m/s); a NACA 0012 airfoil was assumed for the purpose of computing the profile power. Unless otherwise specified, a linear twist distribution of −7.0 deg across the blade span was assumed as a baseline value, and the vortex quadrilateral layout used 20 equally spaced quads spanwise and one chordwise.

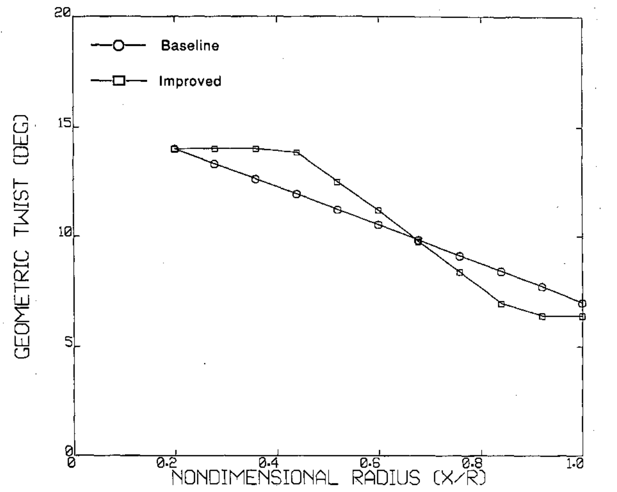

Fig. 2 Baseline and improved twist distribution for a two-bladed rotor at thrust coefficient 0.00394.

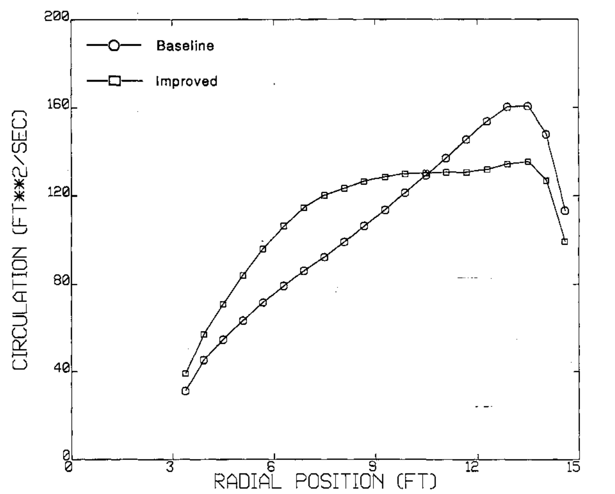

Fig. 3 Baseline and improved bound circulation distribution for a two-bladed rotor at thrust coefficient 0.00394.

The first sample calculations carried out here were very simple problems with limited free wakes. It should be noted at the outset that the objective of these calculations was not to find the global optimum performance but rather to demonstrate that the current procedure could produce design changes that lead to substantial performance improvements at a reasonable computational cost. As will become evident, in the absence of explicit constraints on the total change in twist that is permissible at any given section, very substantial performance improvements can be obtained for the simple problems considered.

The first test involved a case with two free filaments trailing from each blade, one from the blade tip and one from the root. Each featured a single turn of free wake, with prescribed wake used past the point to extend the wake to infinity below the rotor. The baseline rotor was set at a collective pitch corresponding to a moderate thrust coefficient of 0.00394 and the optimization routine was run for 10 loops. Figure 2 shows the initial and final twist distribution on the rotor, whereas Fig. 3 compares the bound circulation distributions. The improved configuration keeps the thrust constant while reducing the power required by 9%, corresponding to an increase in the figure of merit from 0.710 to 0.763.

Clearly, the principal effect of the optimization algorithm is to shift the blade load inboard by increasing the geometric

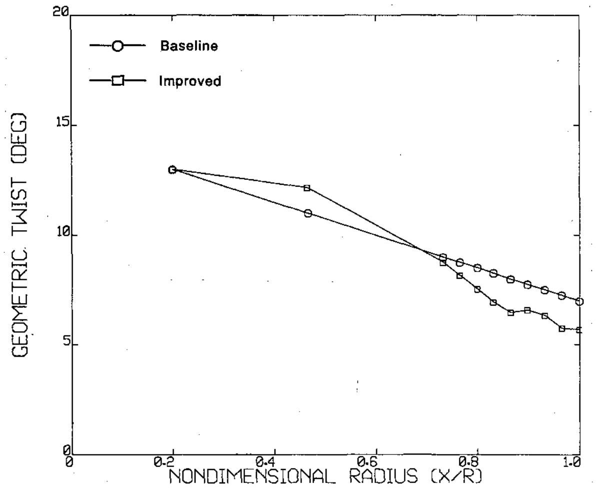

Fig. 4 Baseline and improved twist distribution for a four-bladed rotor at thrust coefficient 0.00574: refined tip mesh.

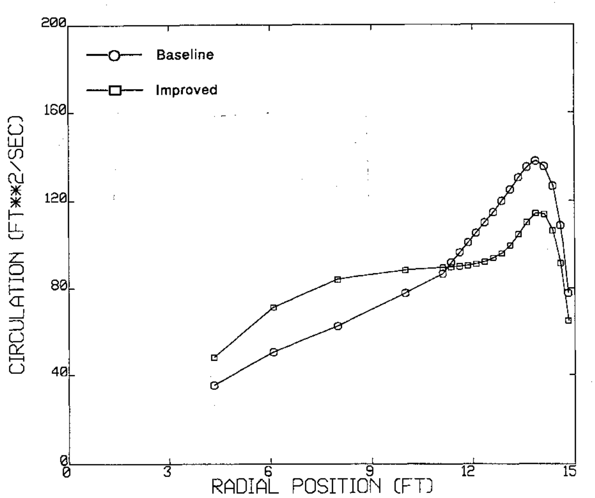

Fig. 5 Baseline and improved bound circulation distribution for a four-bladed rotor at thrust coefficient 0.00574: refined tip mesh.

angle of attack near the root and dropping it outboard, thus producing a more uniform bound circulation distribution. This shifted distribution is apparent in Fig. 3 and is also reflected in a reduction in the induced power because carrying the load inboard reduced the torque required to drive the rotor. (The downwash distribution changes as well, but not enough to counteract the effect of the shift in loading.) The profile power drops only slightly (approximately 1%) as a result of the optimization, and it is clear both here and in subsequent cases that it is the change in induced power that dominates the optimization process.

The case just discussed used 10 spanwise segments, all of equal span, to resolve the twist distribution, and further divided each segment into two vortex quadrilaterals for a total of 20 on the span. This constitutes a relatively coarse resolution of both the downwash distribution and the blade design, and it was of interest to investigate the effect of refining the model used. Also, it was expected that in cases where tip vortices executed reasonably close encounters with following blades that the steep spanwise gradient in downwash due to such close encounters could have an important effect on the blade design. Another set of calculations was thus undertaken using a more refined mesh near the tip. Ten segments were still employed across the blade span, but eight segments (each 0.033R in span, less than half the value of 0.08R used previously) were concentrated outboard, whereas two large segments of span 0.266R were used just outboard of the 20% cutout. Each of these segments featured two spanwise vortex quadrilaterals, which still yielded a total of 20 spanwise for the aerodynamic model.

A four-bladed case was run at moderate thrust (to promote close blade/vortex interaction (BVI) without direct impingement) with this refined mesh, again using 10 loops of the optimization analysis. This run resulted in an increase in the figure of merit from 0.687 to 0.739 at a thrust coefficient of 0.00574. These numerical results are similar to the case discussed earlier, but the details of the final blade design are considerably different. Figure 4 shows the improved twist distribution, which reflects a highly variable twist near the tip. Although the bound circulation shown in Fig. 5 appears similar to previous results, evidently, the twist responds to the highly variable downwash due to the nearby tip vortex (which passes 0.06R beneath the blade at a radius of 0.91R) and must undergo considerable distortion to achieve the minimum induced power in any design step. This sensitivity to local inflow distortions, which depend critically on blade/vortex encounter geometries, emphasizes the importance of retaining a physically accurate free wake treatment in the design optimization problem. It also points out the very important role of the modeling of close BVI on hover performance optimization. It is important to note that the current formulation is best suited for cases where interactions are primarily inviscid; problems involving direct impingement of filaments on rotor blades cannot be handled with confidence.

The test cases examined to this point have all used very simplified two-filament wakes and, thus, have been convenient model problems rather than realistic rotor wake calculations. Past experience with the EHPIC code has suggested that at least four free wake filaments (one representing the rolled-up tip vortex and three large-core vortices modeling the inboard sheet) and two to three turns of free wake are required to obtain meaningful performance results for most conventional rotors (i.e., rotors featuring blades with nearly linear twist and modest taper). Thus, an additonal set of runs was undertaken with the same blade planform just discussed but with a new wake model featuring four filaments and several turns of free wake. The refined blade mesh was retained for these calculations.

In previous cases, it was noted that 10 loops of the optimization routine yielded improvements in performance of roughly five points in figure of merit; such an improvement would represent a dramatic enhancement of the rotor's efficiency. This large improvement was apparently related to the very simple model of the rotor wake model used earlier. The runs with a more realistic four-filament wake produced more modest results; after 10 loops using the same design variable constraints as the two-filament runs, the four-filament case displayed a two-point gain in figure of merit (0.658 to 0.679) at a thrust coefficient of 0.00544, a considerably less dramatic change than noted earlier. The twist and circulation distributions obtained for this case are shown in Figs. 6 and 7, and these reflect considerable differences from the corresponding results in Figs. 4 and 5. The tendency to shift load inboard is less pronounced in the four-filament case, and the rate of increase in improved performance (in terms of the number of cycles required to obtain a given increase in figure of merit for the specified constraints) is considerably less than the more simplified two-filament case. However, the tendency of the blade design to respond to the presence of the tip vortex is still very pronounced.

Another test was undertaken in hover with a practical rotor configuration because it was judged to be of interest to apply this procedure to a known, advanced rotor system to examine the behavior of the design solution. The rotor selected was the square-tip variant of the ATB/XV-15 main rotor. This rotor features three blades with moderate levels of taper and large amounts of nonlinear twist. As before, the blade was divided into 10 segments, each with a piecewise-linear twist distribution. This particular calculation used a wake configuration very similar to the last sample motor run described earlier,

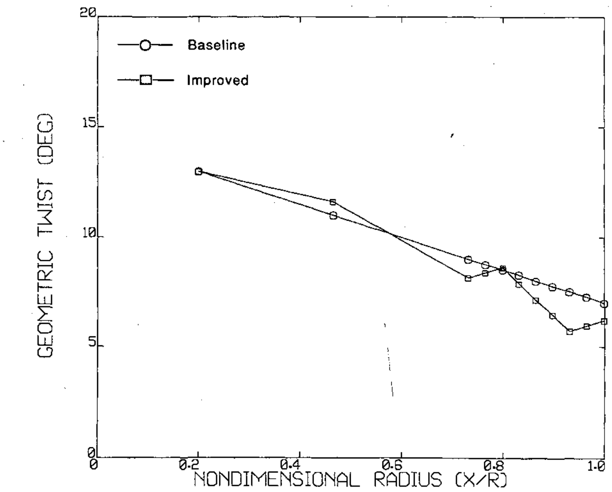

Fig. 6 Baseline and improved twist distribution for a four-bladed rotor at thrust coefficient 0.00544: four-filament wake, refined tip vortex.

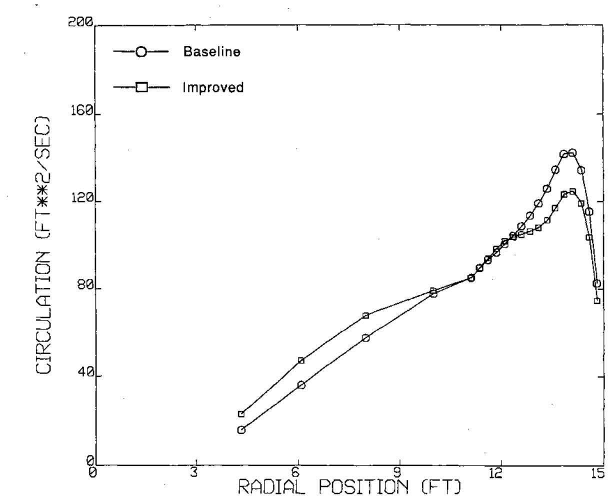

Fig. 7 Baseline and improved bound circulation distribution for a four-bladed rotor at thrust coefficient 0.00544: four-filament wake, refined tip vortex.

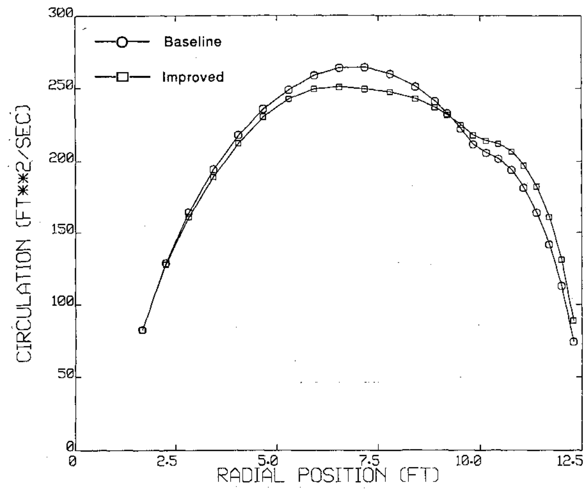

Fig. 8 Baseline and improved bound circulation distribution for the ATB rotor at thrust coefficient 0.0118.

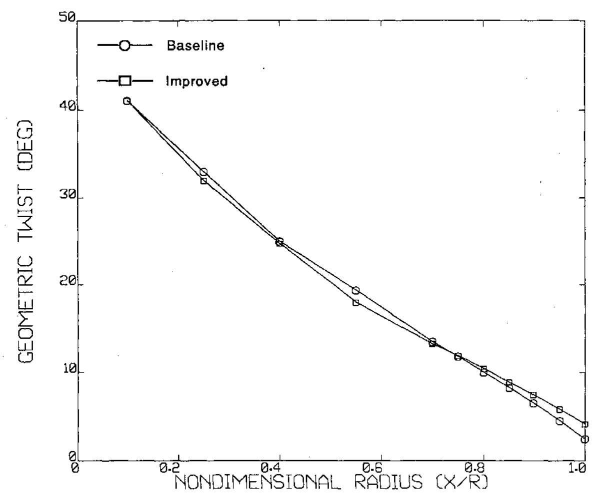

Fig. 9 Baseline and improved twist distribution for the ATB rotor at thrust coefficient 0.0118.

i.e., four free filaments with roughly two to three free turns of wake on each filament. This particular calculation was conducted with a root pitch of 41.0 deg, which yielded a thrust coefficient of 0.0118. Ten design loops increased the figure of merit for this design from 0.797 to 0.818.

Figure 8 shows how the circulation distribution changed during this process, indicating a general trend opposite to that observed earlier with a more conventional low-twist planform because the bound circulation has been shifted outboard. The offloading of the inboard region is also reflected in the alterations in the twist distributions (Fig. 9), which shows decreases in built-in twist inboard and modest increases near the tip. The qualitative difference noted between this case and previous examples—increasing figure of merit by a shift of load outboard instead of inboard—is consistent with the general tendency of the optimizer to drive the blade design to more uniform distributions of circulation along the span. In this case, as in the previous cases, the preponderance of the performance improvement comes about from a reduction in induced power.

Finally, it is of interest to examine the effect of imposing constraints on the changes in twist distribution that are permitted along the blade. The cases discussed earlier have featured inequality constraints that bound the perturbations in twist at any step in the optimization process, but no explicit limits on the total level of twist deviation from the baseline design have been imposed. Such constraints would inevitably be present because of design specifications on the rotor's performance or vibratory loads in forward flight. The current version of the optimization analysis lacks the ability to formally couple the optimization of design for hover and forward flight, but a model problem can be constructed for demonstration purposes.

A linear twist distribution has been used in many rotor blade designs as an acceptable configuration for obtaining reasonable performance across all flight conditions. Assume that such a distribution is used as a baseline and that it is desired to limit deviations from this when altering its design to improve hover performance. The baseline design was taken to be a linearly twisted blade identical to that studied in Figs. 2 and 3, though, here, a four-bladed rotor was assumed. The baseline case featured a twist gradient on the blade of -0.7 deg across each of 10 equally spaced segments. In this constrained calculation, the twist change per segment in the final design was restricted to lie between $+0.5$ and -1.9 deg at the root, whereas the permissible range was $+0.5$ to 1.1 deg at the tip, with a linear tapering of the lower bound assumed between these stations. The tip region was constrained more tightly than the root, as would be the usual practice in an optimization calculation. Also, the constraint on the root twist keeps the blade from generating angles of attack that would produce lift stall; auxiliary calculations indicated that lift coefficients near the root would remain below 0.8 with these constraints, a reasonable level for unstalled performance for the 0012 airfoil assumed in the analysis.

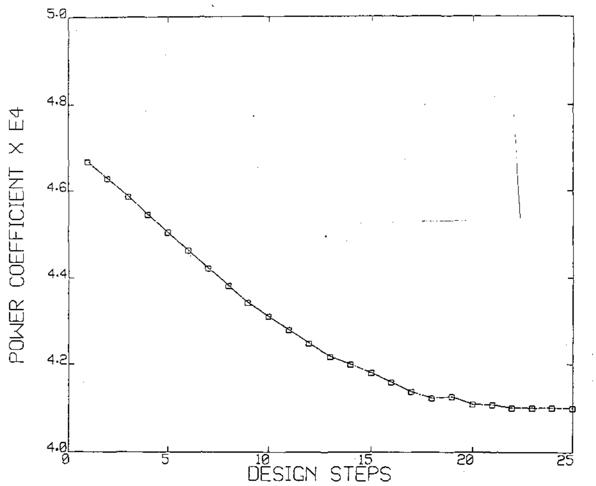

Fig. 10 Progression of rotor power coefficient as a function of the number of design steps for a four-bladed rotor at thrust coefficient 0.00609.

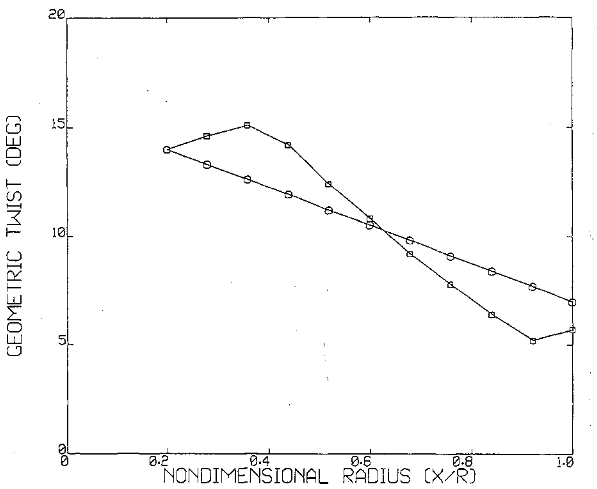

Fig. 11 Baseline and final twist distributions for the case in Fig. 10.

Clearly, constraints would be imposed in a more systematic manner in an actual blade design, but these choices are adequate for the current purpose of demonstrating the operation of the analysis. Figure 10 shows the progression of rotor power required at a constant thrust coefficient of 0.00609 for this case. Note that the constrained case reaches a performance plateau after roughly 20 optimization cycles. The final torque level is 12% below the original value, representing an increase in figure of merit from 0.712 to 0.813. This calculation shows the potential for improvement in the baseline design as well as the ability of this anaysis to find a performance plateau in a properly constrained problem. Figure 11 shows how the final twist distribution for the constrained case compares with the baseline distribution.

The demonstration calculations discussed earlier have focused exclusively on hovering rotors. The rotor design optimization analysis is also applicable to rotors operating in climb. Sample problems similar to those presented here but directed toward optimizing blade twist distributions for climbing flight are discussed in Ref. 22.

Summary

The primary aim of this effort was to establish that a linear optimization algorithm could be successfully coupled to the quasilinear wake relaxation technique that forms the technical foundation of the EHPIC hover performance code. This coupling was indeed found to be possible once a suitable expansion of the matrix of linearized influence coefficients in the coupled blade/wake relaxation was accomplished and new coefficients had been derived for a linearized equation governing perturbations in power (the objective function for the optimization analysis) and the thrust. The derivation of these additional coefficients was simplified considerably because of the preexisting influence coefficients computed in the EHPIC code. The resulting analysis minimizes computationally expensive calls to the original hover relaxation routine, leading to an efficient treatment of a complex problem that retains a refined model of the flowfield.

The results of the various sample calculations discussed earlier demonstrate the success of this coupled algorithm. In several computations, ranging from simplified model problems to more realistic performance calculations, the optimization analysis has identified configurations with substantially improved performance relative to their respective baseline cases. The particular problem chosen for study here, that of twist optimization, regularly led to designs that achieved increases of several points in figure of merit at fixed thrust. Demonstration calculations also showed in prinicple how constraints on the twist distribution can be imposed to bound the design to conform with the requirements of other flight conditions.

This study also pointed out several features that must be added to the analysis described here if a generally applicable performance optimization capability is to be developed. Prominent among these are the ability to include chord, sweep, and other significant planform properties in the analysis; a refined treatment of blade/vortex interaction in the vicinity of the tip, given the importance of such interactions in determining hover performance; improvements in the underlying aerodynamic model of the blade, with an enhanced model of lift stall and near-wake rollup effects on advanced planforms; and the potential requirement for a nonlinear numerical optimization algorithm to address the complex topologies of performance in design space that may arise when several design variables are present in the analysis. Further exploration of the sensitivity of the designs achieved to the refinement of the vortex lattice model of the blade and of the filamentary wake model is also desirable. Such extensions and modifications of the current work may be accommodated readily within the framework developed here to produce a comphrehensive treatment of rotor performance optimization.

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